

P2SF: Physically-based Point Spread Function for digital image processing

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Abstract: P2SF is a tool aimed at estimating the spatially variant point spread function based on the ray-tracing computation of the pupil function. Only 6 parameters are required to describe a monochromatic PSF.

1. Introduction

The optics is an essential part of the imaging system. The optical system is responsible for gathering and focusing the light on the image-sensor. At this stage, the light signal is processed according to the arrangement of the optical elements. For instance, the optics of traditional cameras act as a low-pass filter with respect to the spatial frequency spectrum of the input. Instead, computational cameras exploit novel optical design to treat the light signal in several ways [1]. For the most of imaging systems, the optical processing is the first stage of the image formation (fig 1). Given a point source, the most suitable function to describe such transformation is the point spread function (PSF). In order to provide a complete description of the optical processing, a specific method is needed to gather/estimate all the PSFs ($PSF(d, h, \lambda, x, y)$) collected by the image sensor sampling the entire object space (set of object planes at different distance (d)) for each wavelength. Several image processing techniques need the PSF knowledge in order to enhance image features (e.g. extend depth of field, deblurring, etc.) or recover information (e.g. depth) [1-3]. Therefore, the PSF plays a key role in a large set of image processing applications. An exhaustive and flexible method for treating the optical transformation in a imaging system should include all the aspects related to the PSF: spatially dependent to the point source location, dispersion and diffraction.

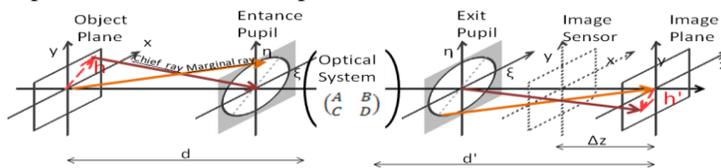


Figure 1 Model of image formation for a generic imaging system

2. State of the art on PSF estimation

For image processing the PSF is usually approached in 3 different ways: model-based approach, experimental measurement and ray-tracing estimation. Model-based PSF can be a valuable solution in iterative optimization, for example, depth estimation [1] and image restoration [2]. On the other hand, this approach does not provide an accurate representation of aberrated optical system due to the lack of physical foundation and the limited parametric description of the PSF, which is based on simple functions, such as pillbox, gaussian, kernel. Instead, experimental measurement of the PSF provides physically-based data[3]. An important shortcoming is the need to repeat the experiment for different set-up (wavelength, point source position) and more memory to store the data. Moreover, the data accuracy is limited by the acquisition devices (image sensor pixel size, noise, etc), and the results are reliable only for the specific design of the imaging system. Any change (e.g. image sensor shift) requires to perform again the overall set of experiments. Finally, a physically-based PSF can be estimated through ray-tracing method, as exploited by Code-V, ZEMAX or Scene3D [4]. Such software need a prior knowledge of the physical parameters of the imaging system components (refractive index, dimension, location, etc.). The ray-tracing methods require a large computational time to perform a single PSF evaluation. Although the PSF can be collected off-line, this increases the memory required to store all the spatially- and wavelength- variant data.

2.The proposed method

P2SF is a novel method aimed at rapidly estimating the overall set of PSFs of an imaging system for focused and quasi- in-focus conditions. For this conditions, a small shift of the image sensor or the point source significantly affects the PSF profile due to the wave nature of light. In order to achieve this goal, P2SF exploits the parametric description of the pupil function ($P(\xi, \eta)$), instead of directly represent the PSF like approached by common model-based methods. In other words, P2SF decomposes and stores the pupil function in terms of wavelength- and spatially- variant parameters. The relation between the $PSF(x, y)$ and the $P(\xi, \eta)$ is reported in section 2.4. The pupil

function, see (1), includes the contribution of aperture shape and light transmission, called apodization function ($A(\xi, \eta)$), and the phase shift introduced by the optics on the spherical wavefront coming from the point source, called wavefront aberration function ($W(\xi, \eta)$).

$$P(\xi, \eta) = A(\xi, \eta) \exp\left(-\frac{i2\pi}{\lambda} W(\xi, \eta)\right) \quad (1)$$

We consider imaging systems with rotationally symmetric components to treat the point source position in terms of depth (d) and field height (h). The optical parameters of the elements are reported as function of the wavelength. Moreover, the $A(\xi, \eta)$ is assumed circular and uniform over the entire pupil. The $W(\xi, \eta)$ is approximated with the primary aberrations, or Seidel aberrations, the 4th order power series polynomial [7]. The first 5 not-zero coefficients ($W_{40}, W_{31}, W_{22}, W_{20}, W_{11}$) reflect specific contributions to the PSF, respectively, spherical aberration, coma, astigmatism, field of curvature, and distortion. A sixth coefficient is introduced to include defocus (W_d). All the coefficients are computed through ray-tracing (i.e. physically-based method) for each λ and normalized to λ . We consider the PSFs for which the defocus coefficients is less than 10λ in order to satisfy in focus and quasi-in-focus imaging. Beyond this limit the wave phenomena become less relevant than geometrical spread. For a given imaging system, we distinguish and sequentially estimate the following features: shift-invariant (SIFs), which do not change moving the point source; quasi-invariant (qSIFs), do not change significantly moving the point source (in general they are constant in sub-region of the object space); and spatially variant (SVFs), which change significantly by slightly moving the point source. Such a distinction is not valid for the chromatic dependence, so SIFs, qSIFs and SVFs have to be evaluated at each λ .

2.1 Shift invariant features

All the surfaces are analyzed through Gaussian Optics to calculate ABCD matrix, which represents the equivalent transfer function for the entire optical system in paraxial approximation. This matrix is computed by considering the overall sequence of refractions and translations occurring to a ray travelling from the first surface to the last surface of the optical system. In paraxial approximation such transformations are described with a 2×2 matrix. The ABCD matrix is given by the backward matrix multiplication and allows to estimate the cardinal points (principal points, nodal points and focal points) and to locate the gaussian image point. Considering fig. 1, d' and h' are obtained by combing the matrix coefficients and point source position (d, h) [5].

2.2 Quasi-shift invariant features

For quasi-invariant features we refer to the entrance (EnP) and exit pupil (ExP). The knowledge of the pupils' location and dimension are essential to compute the pupil function $P(\xi, \eta)$ and the numerical aperture (NA). The EnP and ExP are, respectively, the backward and the forward gaussian images of the aperture stop (ApS) of the system, which is the aperture limiting the angle of acceptance of the light coming from the point source [6]. From the image formation point of view the wavefront delivered by the point source and 'accepted' by the EnP seems to emerge from the ExP with a phase shift equal to the wavefront aberration function.

2.3 Spatially variant features

Any (d, h) combination corresponds to a different pupil function and, then, a different PSF. We assume a uniform transmission of the light over the aperture (i.e. $A(\xi, \eta)$ is a pillbox function) and a $W(\xi, \eta)$ defined with 5 Seidel coefficients and one more for the defocus. Consequently, the SVFs consist of these 6 coefficients. W_d is equal to $(NA^2 \Delta z)/(2\lambda)$, with Δz the distance along the optical axis between the Gaussian image point and the image sensor. The 5 Seidel coefficients are computed through the Seidel-sum method [7] based on the paraxial ray-tracing of marginal and chief ray (see fig.1) which travel, respectively, the longest and the shortest optical path from the object to the image space. These rays depend on the location of the point source and the related EnP and ExP. A database of SVF is gathered by iteratively changing the point source (d, h) and the wavelength (λ).

2.4 The Rendering of the Point Spread Function

The PSF is related to the $P(\xi, \eta)$ through the *rendering stage*. Starting from the 6 coefficients (SVFs) describing a unique $P(\xi, \eta)$, the PSF can be rendered with as many samples as the application requires. For a position (d, h) of the point source not present in the database, each coefficient is estimated by interpolating the closest data in terms of change in depth (Δd) and field height (Δh) according to table 1.

Primary aberration	Spherical aber.	Coma	Astigmatism	Field of curv.	Distortion	Defocus
Coefficient	W_{40}	W_{31}	W_{22}	W_{20}	W_{11}	
Δd	inverse prop.	linear	Quadratic	Quadratic	quadratic	inverse prop
Δh	constant	linear	Quadratic	Quadratic	cubic	constant

Table 1: Interpolation of the spatially-variant dataset.

According to [6] for incoherent light the PSF can be compute as following:

$$PSF(x, y) = |FT\{P(\xi_s, \eta_s)\}|^2 \quad (2)$$

The scaling factor s is equal to λ/NA . The user could specify the resolution and the number of sample (N_{PSF}) for the PSF. The number of sample for the pupil function (N_{PUPIL}) is, at least, 2 times N_{PSF} to satisfy the Nyquist limit. In addition, the N_{PSF} has to be a multiple of 2 in order to exploit the Fast Fourier Transform (FFT).

3. Result and discussion

P2SF has been implemented in MATLAB code and tested on a 64bit –Window 7 machine (Platform: quad-core intel i5 processor @ 2.2 GHz @ 4GB RAM). The following tests are aimed at evaluating P2SF for wavelength- and spatially- dependence. The wavelength dependent optical properties of the system components lead to repeat the entire estimation process. This is usually valid for all the methods : ray-tracing, model-based, and P2SF. Figure 2 reports the computational time (CT) required by each P2SF's stage (SIFs, qSIFs and SVFs) depending on the number of refractive elements (#RE). We use a dataset of optical systems available from [9]. While CT is almost constant for SIFs and SVFs, it increases for qSIFs due to the iterative method used to detect which component is the ApS and, then, find the EnP and ExP [6]. According to the specific imaging application, this result could be used to determine how many wavelengths can be considered and whether the off-line computation and storage are needed. Instead , the rendering stage always requires on-line computation. Considering the complexity of FFT (proportional to $N_{PUPIL} \log_2(N_{PUPIL})$) and the necessity of $N_{PUPIL} \geq 2 N_{PSF}$, a specific application analysis is necessary to determine the most suitable parameter trade-off for the rendering stage. Finally, for testing the spatially variant PSF we considered a Double Gauss 28° FOV with in-focus source placed at infinity for three different height field (0°, 10°, 14°) at $\lambda=656.3$ nm . Data (32x32 points in a 22.12 x 22.12 μm area) generated with HuygensPSF method (i.e. ray-tracing method) though ZEMAX is used as reference. We compare the results of P2SF with a 2D-Gaussian (2dG) function (i.e. model-based method) fitted on the reference data. Figure 2 reports the R-square (%) of P2SF and 2dG with the HuygenPSF. For on-axis position both methods show good statistical significance. For off-axis position, while the capability of prediction for Gaussian function decreases significantly, P2SF is stable. This is due to the asymmetry induced on the PSF profile by the coma and astigmatism. Although the P2SF is not able to replicate HuygenPSF, the proposed method allows to provide the result in less than ~ 500 ms versus several minutes for HuygenPSF . Additional aberration coefficients could be included to improve accuracy.

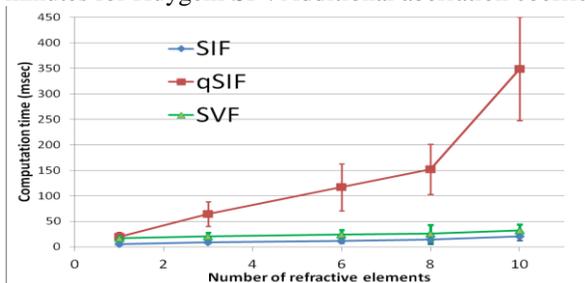


Figure 1 P2SF: Computation time for each stage of the feature estimation

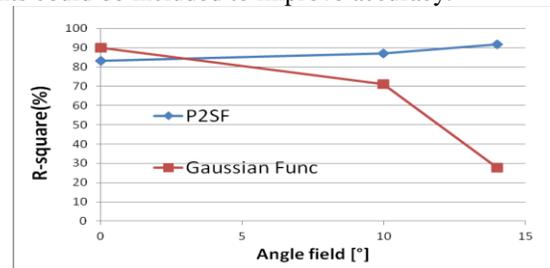


Figure 2 R-square for P2SF and Gaussian function (fitted) based on the data generated though HuygenPSF method

4. Conclusion

We propose a tool able to rapidly estimate the pupil function and the PSF by specifying the properties of the imaging system and point source. P2SF is both model- and physically- based . The P2SF can be customized for specific image processing to treat accurately and rapidly the wavelength- and spatially- dependence of the PSF.

4. References

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